

# N維球體的一個奇妙性質

## On a Curious Property of N-dimensional Sphere

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**摘要：**在N維球體與其邊界之間有一個奇妙的性質。首先，觀察有趣的現象，圓盤的面積對於半徑的導數等於其圓週長，類似地，球的體積對於半徑的導數等於其表面積，最後，證明奇妙的性質，N維球體體積對於半徑的導數等於其N維球面積。

**關鍵詞：**圓盤、圓週長、球體體積、球面積、面積分、三角代換法

**Abstract:** There is a curious property between n-dimensional sphere and its boundary. It is interesting that the derivative of the disk area with respect to radius is equal to its circumference. Similarly, the derivative of the spherical volume with respect to radius is equal to its spherical area. At last, it is proved that the derivative of the n-dimensional spherical volume with respect to radius is equal to its n-dimensional spherical area.

**Key words:** disk, circumference, spherical volume, spherical area, surface integral, trigonometric substitution

## 1. INTRODUCTION

This article will show a curious property between n-dimensional sphere and its boundary. The n-dimensional sphere  $S_n$  ( $n > 1$ ) with radius  $r$  is defined to be the set  $S_n = \{(x_1, \dots, x_n) | x_1^2 + \dots + x_n^2 \leq r^2\}$ . Therefore the boundary  $\partial S_n$  of  $S_n$  should be the set